Mass of Jupiter – Advanced Module

Program overview *Lessons needed: Flexible, but at least two separate lessons will be needed* This program guides teachers and students through calculating the mass of Jupiter. It looks at Kepler's third law of planetary motion, using SPIRIT to image Jupiter and calculate Jupiter's mass. The workflow requires some algebra and trigonometry, including the use of Pythagorean's theorem. Students may need prior knowledge, or time to learn, Pythagorean theorem and the order of operations. This program is presented as an entire workflow and can be completed in a timeframe that suits; however, you will need to allow for at least two lessons as time is needed to take the images of Jupiter using a SPIRIT telescope, either in live viewing or scheduler. We will be using https://www.wolframalpha.com, a computational website to complete our calculations. Please see 'extra activities' at the end of program for activities that may be suitable for before, during and after this unit of work. It is assumed teachers have a background knowledge of using Stellarium and SPIRIT to guide their students through it.

Skills focus:	**Required digital resources:**

• Trigonometry
• Kepler's laws of planetary motion
• STEM skills
  ○ Problem solving
  ○ Critical analysis
  ○ Independent thinking
  ○ Digital literacy
	Device (laptop, computer, tablet) with internet access
	Stellarium – (free software) http://stellarium.org
	A FTP program (recommended free software Filezilla https://filezilla-project.org)
	FITS liberator – (free software- converts FITS files to TIF files to use in photoshop)
	https://noirlab.edu/public/products/fitsliberator/

Curriculum links:

Science	***Maths***
ATAR Physics- **year 12,** unit 3- Gravity and electromagnetism	Investigate Pythagoras' Theorem and its application to solving simple problems involving right-angled triangles (ACMMG222) **Year 9**

<table>
<tr>
<td colspan="2">Scientific knowledge has changed peoples' understanding of the world and is refined as new evidence becomes available (ACSHE134 and ACSHE119) Year 7 and 8
Scientific understanding, including models and theories, is contestable and is refined over time through a process of review by the scientific community (ACSHE157 and ACSHE191) Year 9 and 10

Advances in scientific understanding often rely on developments in technology and technological advances are often linked to scientific discoveries (ACSHE158 and ACSHE192) Year 9 and 10</td>
<td>Apply trigonometry to solve right-angled triangle problems (ACMMG224) Year 9

Solve right-angled triangle problems including those involving direction and angles of elevation and depression (ACMMG245) Year 10

Substitute values into formulas to determine an unknown (ACMNA234) Year 10

Solve simple trigonometric equations (ACMMG275) Year 10A

Apply Pythagoras' Theorem and trigonometry to solving three-dimensional problems in right-angled triangles (ACMMG276) Year 10A</td>
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<tr>
<td colspan="2">Science Inquiry Skills – year 7-10
• Planning and Conducting
• Processing and Analysing Data and Information
• Evaluating
• Communicating
Digital Technologies – year 7 – 10
• Collecting, managing, and analysing data
• Digital implementation
• Creating solutions</td>
<td>Numeracy Proficiency Strands
• Understanding
• Fluency
• Problem-solving
• Reasoning
General capabilities:
• Numeracy
• ICT capabilities
• Critical and creative thinking</td>
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<tr>
<td>Lesson 1 Prequisites:
• Internet enabled devices
• Mass of Jupiter workflow sheet</td>
<td colspan="2">1. Kepler's 3rd law of planetary motion is what we will be using to determine the mass of the Sun and Jupiter. A video explaining this is more detail can be found here.
The law we are going to be using looks like:
$$P^2=\frac{4\pi^2}{G}\left(\frac{a^3}{M_1+M_2}\right)$$</td>
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</table>

photocopied for students Teachers should have some experience using Stellarium and SPIRIT to help students	This can look confusing but when you break it down it is easier to understand. $4\pi^2$ = a known number. (4x3.14^2) G= another known number. It is known as Newton's gravitational constant. More on that can be found here. M_1+M_2= the mass of two objects. E.g. the Sun and Earth or Jupiter and Europa. The mass of Europa is so insignificant when compared to Jupiter that we can just use M_1. a= The semimajor axis. The distance the orbiting object is from the other mass. For example, the distance Europa is from Jupiter. P= the orbital period of the orbiting object. In this case, how long it takes Europa to orbit Jupiter. However, we don't want to know the orbital period of an object but rather the mass of an object so let's rewrite the equation: $$M_1 = \frac{4\pi^2(a^{3)}}{G(P^2)}$$ To work out this equation we need to know what *a* and *P* equal as we already know $4\pi^2$ and *G*.
Lesson 2 Prerequisites: • Internet enabled devices • Stellarium downloaded • If using; an FTP program • FITS liberator downloaded Teachers will need to check that the Jupiter images have been taken	2. Let's practice using this equation to figure out the mass of the Sun. $$M_{Sun} + M_{Earth} = \frac{4\pi^2(a^{3)}}{G(P^2)}$$ The mass of the Earth is tiny and insignificant compared to the mass of the Sun so we can reasonably represent it as: $$M_{sun} = \frac{4\pi^2(a^{3)}}{G(P^2)}$$ We know what $4\pi^2$ and *G* are so now we need to know *a*, (the distance between the orbiting object and the mass) and *P* (the length of time the orbit takes).

With Earth and the Sun, we know *a*= 1 AU and *P*= 1 year so we can now work out the equation as:

$$M_{sun} = \frac{4\pi^2(1\ AU^{3)}}{G(1\ year^2)}$$

Use https://www.wolframalpha.com , a computational intelligence website to work out the answer.

When you type your equation into wolframalpha it should look like:

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(4*pi^2*(1 AU)^3)/(G*(1 year)^2)
```

Pay attention to the placement of brackets.
The answer from wolframalpha is 1.991×10^{30} kg, which is correct. (within an acceptable range of error)

3. Now students know how to use Kepler's third law to calculate mass we are going to do the same for Jupiter.

One of the SPIRIT telescopes, needs to be set up to take an image of Jupiter and its Galilean moons. Use SPIRIT to get images by:

a) *Live viewing- If you are using live viewing and would like students to create a plan to practice their coding skills use the information here.*

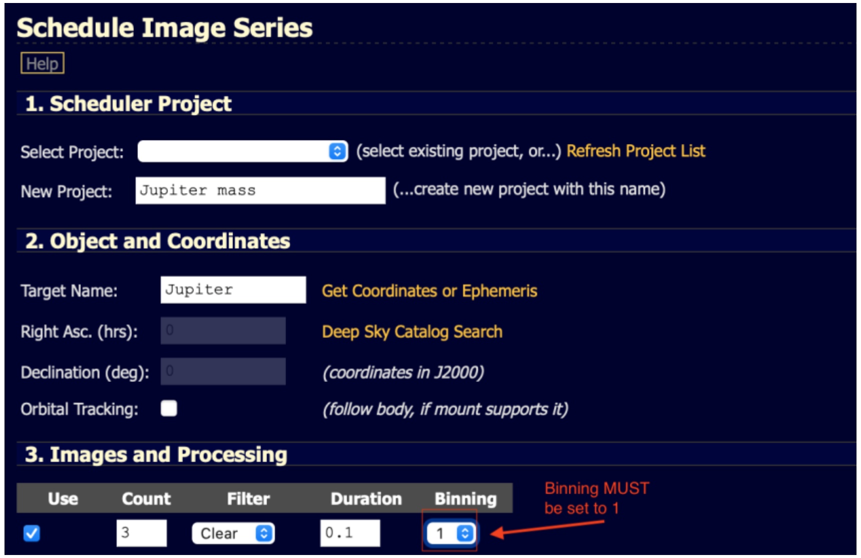
Please note: If using live viewing teachers need to book the appropriate time on SPIRIT 2. Students or teachers will need to log in at the requested time to complete their viewing plan and live viewing.

b) *Scheduling- If you are using the scheduler then students should follow the instructions here.*

Please note: Students or teachers will need to include an email address in the schedule browser section of the web interface to make sure they get notified when the images are ready.

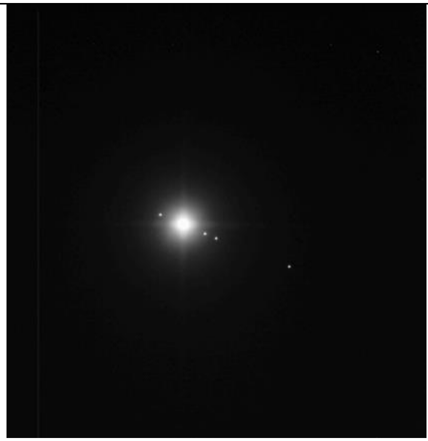
As Jupiter is a very bright object that is close to the telescopes, the exposure time should be set to ***0.1 seconds and binning MUST be set to 1.***

An example of this set up as a viewing project on SPIRIT 4 is shown below.



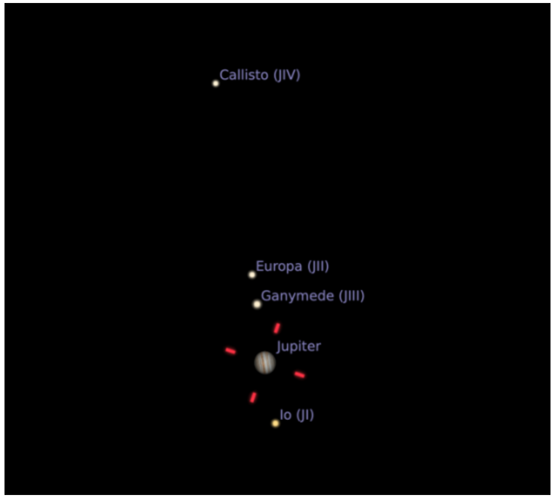
4. Once your images have been taken, access them using an FTP program or download directly from the web interface. Information on how to use Filezilla can be found here.

The images may look very blown out to begin with.



However, once you have used Fits Liberator to convert the images to tiff files, you will be able to see Jupiter and its’ moons clearly.

Information on using Fits Liberator to convert your files can be found here.

	5. Using your SPIRIT image of Jupiter and the Galilean moons, work out which of the moons is Europa. You can do this by setting Stellarium to the exact time and date your image was taken, searching for Jupiter and zooming in. (You can work out the exact time by viewing the logs of your image. The time on the logs show UTC time, so you need to add 8 hours to get the time in Western Australia) Note: You may need to flip your picture so that it matches what you see on Stellarium. See here for more information.  6. Once you know which moon is Europa, we can calculate the value of *a* using trigonometry.

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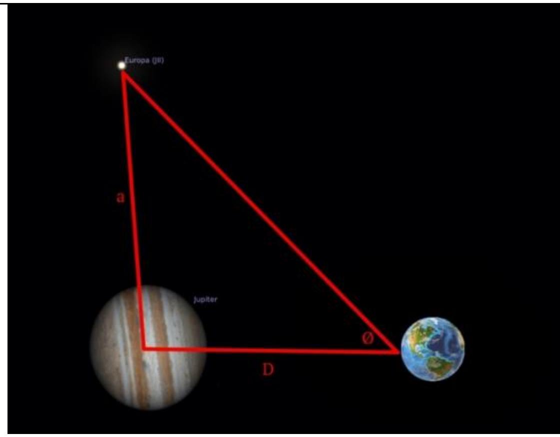


In order to calculate *a* we need to use the small angle formula (More information can be found here):

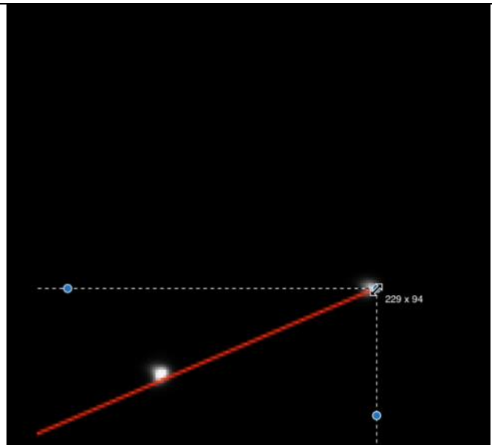
$$\emptyset = \frac{a}{D}$$

We can work out Ø by measuring the angular separation. Start by using Preview (on Mac) or MS Paint (on windows) to open your image.

The images below are using Preview on Mac.

Draw a line from the centre of Jupiter to the centre of Europa. This may take some practice.

Using your mouse, create a selection rectangle of your line, and count the amount of pixels **<u>across</u>** and **<u>down</u>**.
</td>
</tr>
</table>



Please note the following figures are examples only.
You will need your own SPIRIT image of Jupiter to complete the calculations

7. In my example the box is 229x94 pixels, your numbers may be different.
We want the hypotenuse of the triangle we just created. We need to use the Pythagorean theorem to calculate that (more information on the Pythagorean theorem here).

In this example: $\sqrt{(229^2 + 94^2)}$

Use wolphramalpha.com to calculate.

$$\sqrt{(229^2 + 94^2)} = 247.5419156425836501\ pixels$$

Now we know that, we need to convert the pixels into radians.
Use the following table to convert your pixels into arcseconds first:

Telescope	Arcseconds per pixel **(based on binning 1)**
SPIRIT 2	0.63
SPIRIT4	0.7
SPIRIT 6	0.83

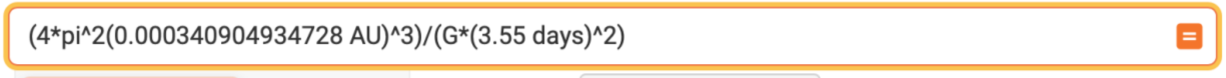
The above image was taken on SPIRIT 4, so to calculate the arcseconds:

$$247.5419156425836501 \times 0.7 = 173.279340949808555\ arcseconds$$

Now convert from arcseconds to radians. There are 206,265 arcseconds in each radian.

	$$\frac{173.279340949808555}{206265} = 0.00084008116\, radians$$ Now I know ø=$0.00084008116\, radians$ and $$\emptyset = \frac{a}{D}$$ Therefore $0.00084008116 = \frac{a}{D}$ and $a = 0.00084008116 \times D$ To work out D, I need to know the date my picture was taken. The example picture was taken on 7th September 2021. When I type in 'distance to Jupiter September 7th 2021' on wolframalpha, I learn that D= 4.058 AU So $a = 0.00084008116 \times 4.058\, AU$ $a = 0.00340904934728\, AU$ 8. P=3.55 days This is the amount of time it takes Europa to orbit Jupiter. This can be confirmed by watching and timing Europa's orbit on Stellarium. 9. Let's look back at Kepler's third law: $$M_1 = \frac{4\pi^2(a^{3)}}{G(P^2)}$$ We now know that a=0.0.00340904934728 AU and P=3.55 days. $$M_1 = \frac{4\pi^2(0.00340904934728\, AU^{3)}}{G(3.55\, days^2)}$$

The above equation typed into wolframalpha looks like:



10. Check your answer against the real answer **(1.898×10^{27} kg)** and see how close you are.

Discussion points:

- Why don't the answers match exactly?
  Accuracy of tools, accuracy of counting pixels, error in processing image etc.
- How different is your answer from the correct answer?
  Different for each student
- What is an acceptable margin of error?
  To be discussed and agreed upon as a class, usually within one power of 10.
- What could you change to make your answer more accurate?
  how the image is processed, drawing the line between Jupiter and Europa, using more than one image to verify, using more decimal places
- What other uses might Kepler's 3rd Law have?
  Calculating mass, distance or orbital periods of other celestial objects

Extra activities:

a) Investigate Johannes Kepler and Isaac Newton. Specifically, how their discoveries about the universe supported each other. How did they build on each other's' ideas?
b) Create a timeline demonstrating our understanding of gravity and how it has changed and grown over time
c) Create a chart comparing the four Galilean moons. E.g., their sizes, their composition, the surface, temperature, orbital period

What next:

To extend the same maths concepts students can complete the activity 'weighing a galaxy' found here: https://www.icrar.org/outreach-education/resources/ This covers the trigonometry concepts from this program in a different astronomy context, and incorporates radio astronomy.

Using SPIRIT, students can undertake more astronomy research projects including photometry.

If you are looking for ideas or support on how to use SPIRIT in your classroom, please contact us at any time at: spirit@icrar.org